Geostatistical models – Part II

Applied Spatial Statistics

Spatial linear models

Part I introduced the standard model

$$Y_i = \mu_i + Z_i + \varepsilon_i$$

- The mean is the same as linear regression

$$\mu_i = \beta_0 + X_{i1}\beta_1 + ... + X_{ip}\beta_p$$

- There are two error terms:
  - Z_i is spatially-correlated
  - ε_i are independent across i

In Part II we will discuss models for Z_i

Spatial covariance models

- The Z_i capture spatial correlation not explained by the covariates
- Assume they are Gaussian with mean zero and variance σ^2
- Define covariance function as

$$\text{Cov}(Z_i, Z_j) = C(\mathbf{s}_i, \mathbf{s}_j) = \sigma^2 \rho(\mathbf{s}_i, \mathbf{s}_j)$$

- The spatial correlation function ρ is large when $\mathbf{s}_i$ and $\mathbf{s}_j$ are close and small when they are far apart
- In the extreme, assume that $\rho(\mathbf{s}, \mathbf{s}) = 1$ so that Z is a continuous function of $\mathbf{s}$

Spatial covariance models

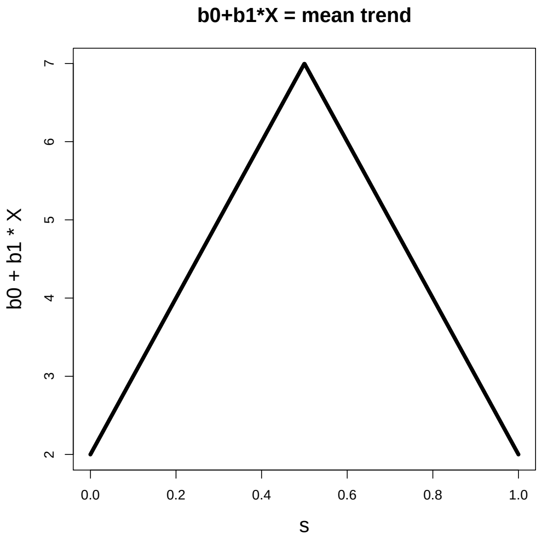
Common sources of spatial variation:

- Missing covariates that have spatial patterns
- Shared exposure to a common point sources (power plant)
- Share exposure to meteorological events
- Dispersion (e.g., pests, air, water)
- Measurement error, e.g., blurred satellite images

Fake data in one dimension (say s=longitude)

Model: $Y_i = \beta_0 + \beta_1 X_i + Z_i + E_i$

Fake data in one dimension (say s=longitude)



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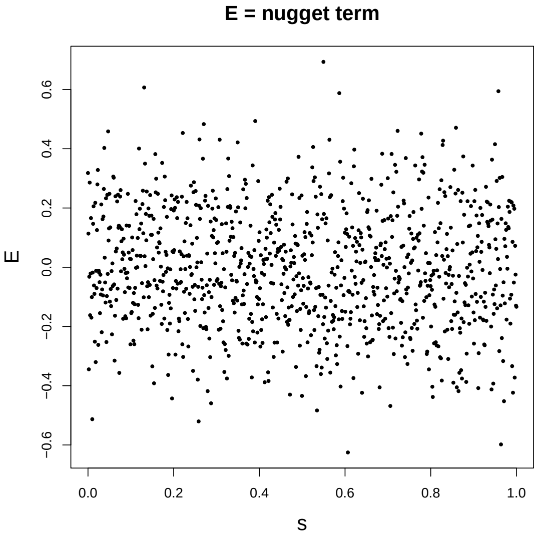

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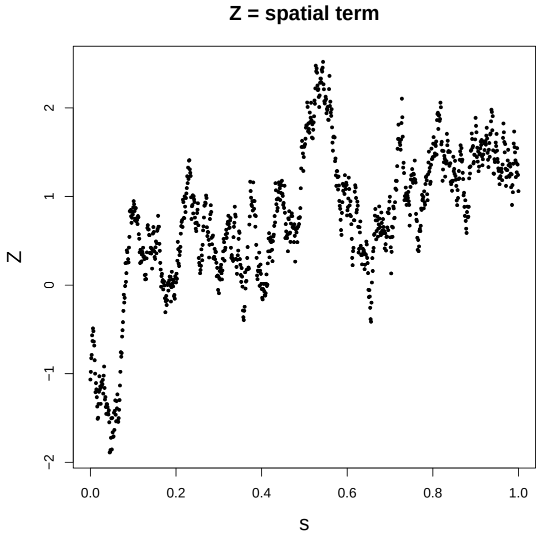

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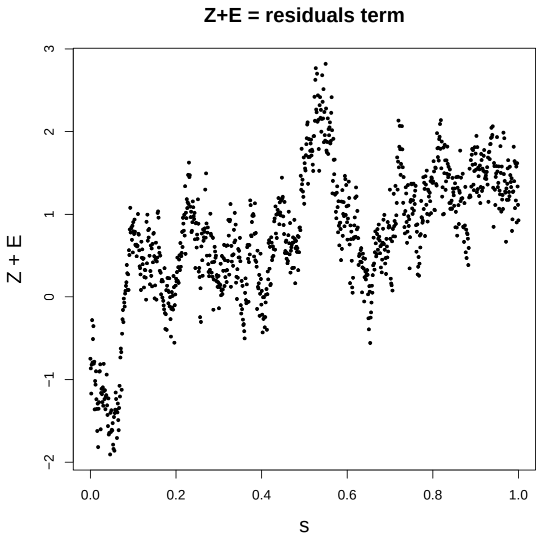

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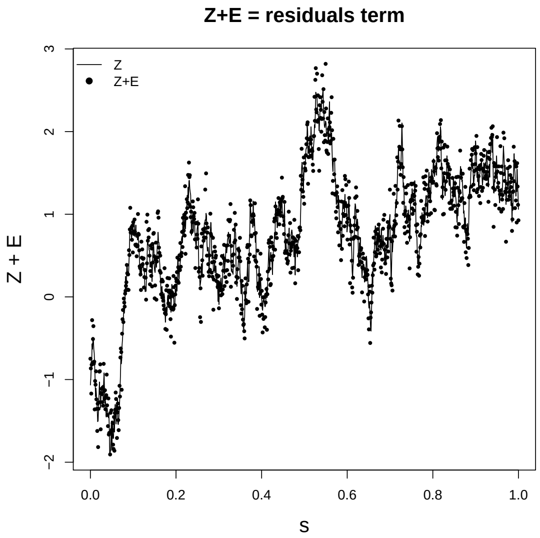

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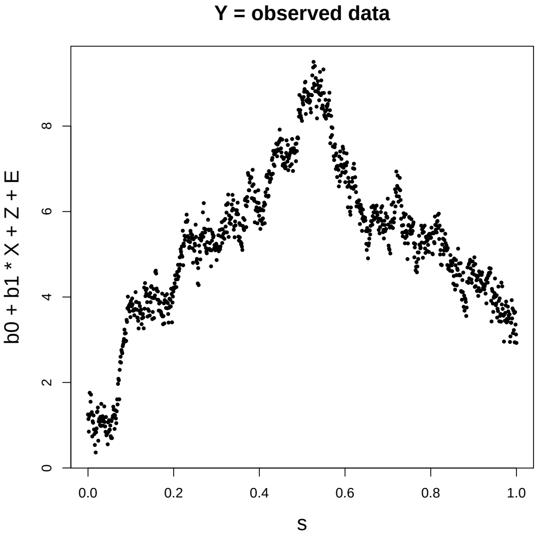

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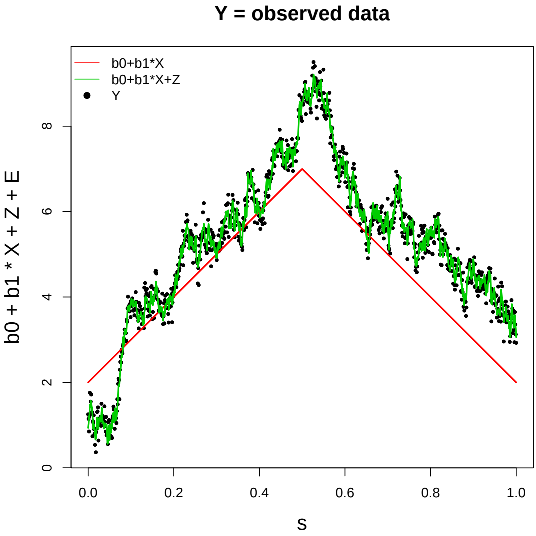

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Spatial covariance models

Which factors might contribute each type of variation in the air pollution example?

- Mean trend:
- Spatial error:
- Nugget error:

Spatial covariance models - isotropy

- The spatial correlation term $\rho(\mathbf{s}_i, \mathbf{s}_j)$ is the novel term in a spatial analysis
- As illustrated from the plots above, it is challenging to estimate
- For example, how would you estimate $\rho(0.2, 0.3)$?
- We will make several simplifying assumptions (isotropic, stationary, etc)
- We will then propose models for the correlation function that satisfy these assumptions

Spatial covariance models - isotropy

- The simplest assumption is **isotropy**
- A covariance is isotropic if it depends only on the distance between locations
- Denoting the distance between $\mathbf{s}_i$ and $\mathbf{s}_j$ as d_{ij}, then

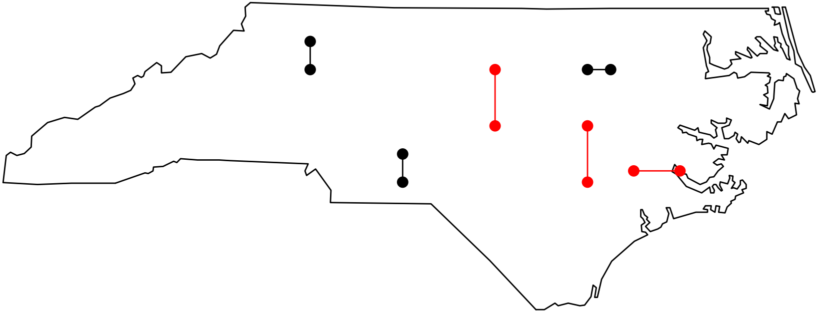
$$\text{Cor}(Z_i, Z_j) = \rho(\mathbf{s}_i, \mathbf{s}_j) = \rho(d_{ij})$$

- For example, the exponential correlation model is

$$\rho(d) = \exp(-d/\phi)$$

where ϕ determines the range of correlation

Spatial covariance models - isotropy



Pairs with the same color have the same correlation

Spatial covariance models - stationary

- Stationarity is more general than isotropy
- A covariance is stationary if it depends only on the difference $\mathbf{s}_i - \mathbf{s}_j$

$$\text{Cov}(Z_i, Z_j) = \rho(\mathbf{s}_i, \mathbf{s}_j) = \rho(\mathbf{s}_i - \mathbf{s}_j)$$

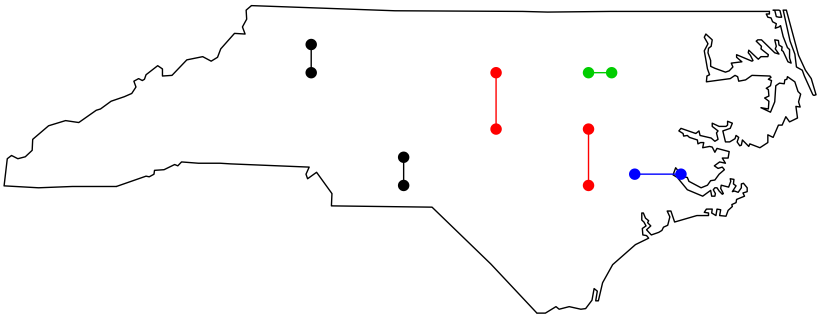
- Stationary means that the covariance is the same throughout the spatial domain
- For if two sites in the east have $\mathbf{s}_1 - \mathbf{s}_2 = \mathbf{h}$ and two sites in the west have $\mathbf{s}_3 - \mathbf{s}_4 = \mathbf{h}$, then

$$\text{Cor}(Z_1, Z_2) = \text{Cor}(Z_3, Z_4)$$

Spatial covariance models - anisotropy

- Stationary means that the covariance is the same throughout the spatial domain
- However, a stationary covariance can depend on both the distance between locations and the angle
- A covariance that is depends on the angle between location is called **anisotropic**
- For example, if the predominant wind pattern is east/west, then east/west pairs may have stronger correlation than north/south pairs

Spatial covariance models - anisotropy



Pairs with the same color have the same correlation

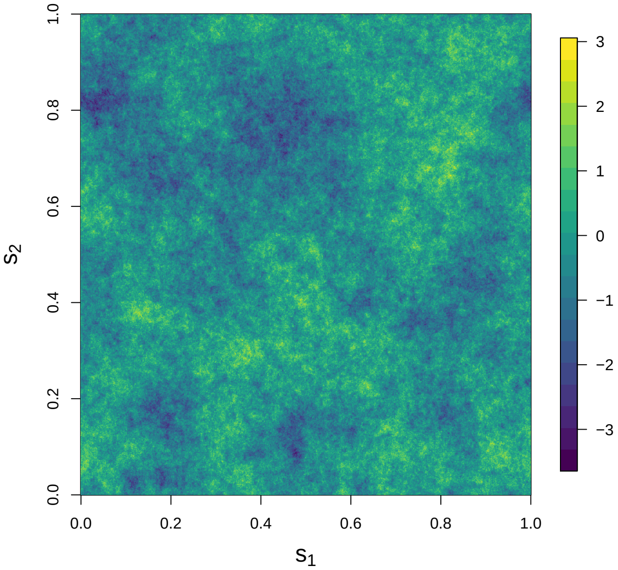
Spatial covariance models - nonstationary

If the spatial covariance can change in different parts of the spatial domain, it is **nonstationary**

- Stronger correlation in the east than the west
- Larger variance in the north than the south
- Stronger correlation between sites at the same elevation
- Larger variance in cities than rural areas

Nonstationary covariance are much harder to estimate

Isotropic, anisotropic or nonstationary?



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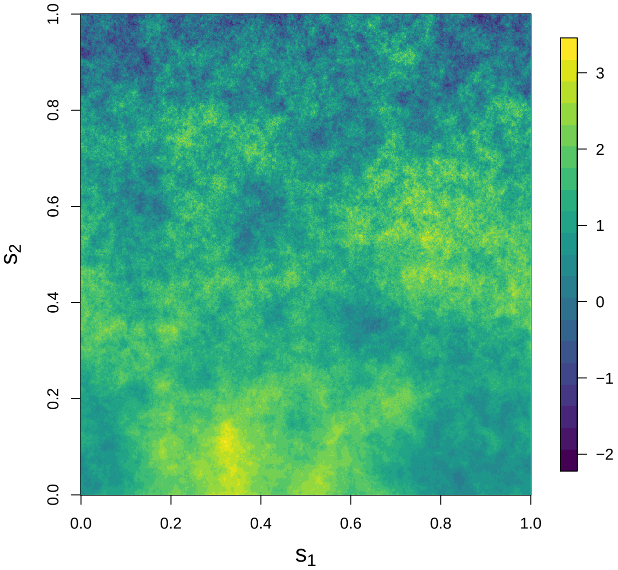

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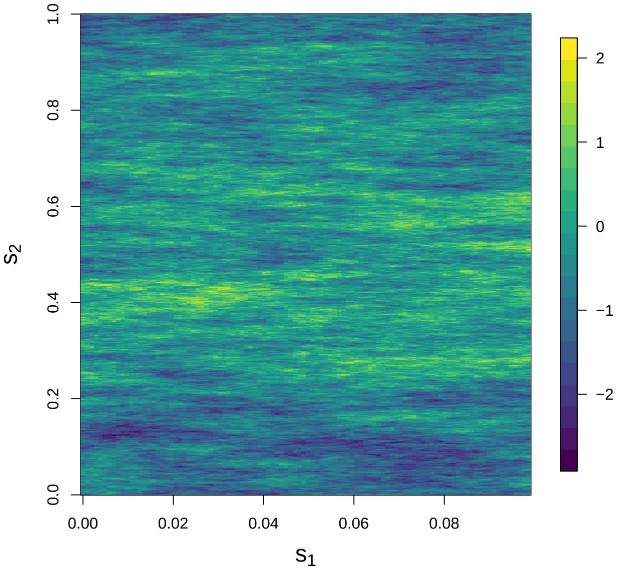

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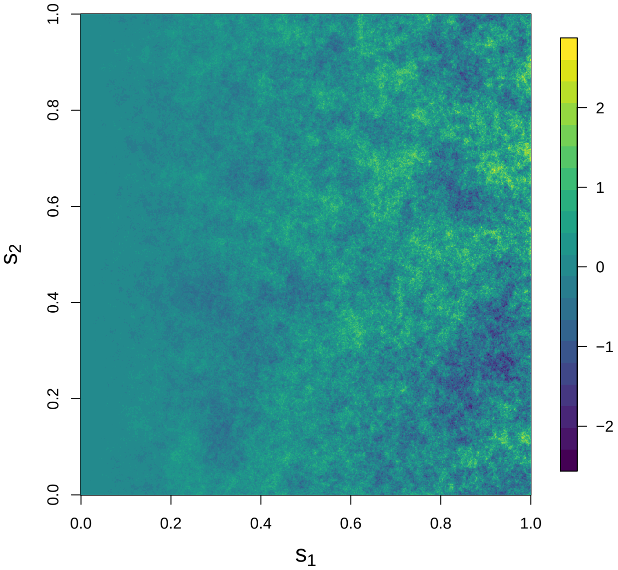

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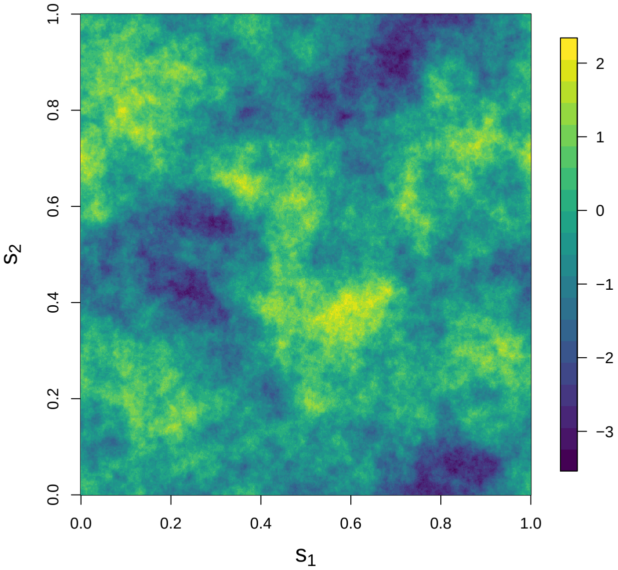

Isotropic, anisotropic or nonstationary?

- Landsat data
- Malaria data
- Link to maps of the microbiome data